The number of π's so far in math is 3892

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Introduction

There are 3892 publications indexed in MathSciNet with Π or π in the title, up to March 08, 2024. Among these 3892 publications, 3384 of them are journal articles, 452 are book collection articles, 45 are books, and there are 5 book collections and 5 theses with Π or π in the title.

We present various perspectives of these 3892 publications in MathSciNet, including a summary of the main theorems of the oldest π-papers, an introduction to the mostly-cited π-publications, and an analysis of the most uses of π in terms of their Primary Classifications (MSC2020).

Oldest π-papers are written in German

The six oldest papers dealing with Π or π are all written in German, in which π is understood in the naive way of the irrational number $\pi = 3.14159...$, and $\Pi(x, y)$ denotes an arbitrary rational function for which there is a general second-order equation between x and y.

The oldest paper ***Über den Ausdruck** $\pi = \frac{2}{i}\log(i)$* was written by **Karl Schellbach** (1832). The title translates to English as *About the expression* $\pi = \frac{2}{i}\log(i)$. Unfortunately, since this 3-pages paper is unlicensed, we do not know if Schellbach proves anything more than the identity in the title.

Ferdinand von Lindemann's famous proof that π is a transcendental number is the fifth oldest (1882): ***Ueber die Zahl π***, or, *About the number* π, where Lindemann used Hermite's result that e is transcendental.

The first π-paper in English is by **E. Hastings Moore** (1894), where he proved that one can define $f(z) = \sin(\pi z)/\pi$ by systems of characteristic functional properties.

Early mathematicians mostly think of π as a real constant. However, in 1924 **Cornelius Gouwens** (who is a student of Leonard Dickson) proved a system of polynomial invariants with integer coefficients of the linear group modulo $\pi = p_1^{\lambda_1} p_2^{\lambda_2} \cdots p_n^{\lambda_n}$, a product of prime numbers. Later in 1933, **Stanley Skewes** gave assuming the truth of the Riemann hypothesis the first Skewes' number, which by definition is any of the large numbers before which $P(x) := \pi(x) - \mathrm{li}(x)$ has changed signs. Here $\pi(x)$ is the number of primes less than x, and $\mathrm{li}(x) = \int_0^x \frac{dx}{\log x}$ is the logarithmic integral.

Mostly-cited π-publication is a TCS book

The book ***The π-calculus: a theory of mobile processes*** by theoretical computer scientists **Davide Sangiorgi** and **David Walker** (2001) is the mostly-cited π-publication (with 228 citations according to MathSciNet). As Andrea Maggiolo-Schettini described in the Mathematical Review:

> *[...] Mobility [...] is the links that move in an abstract space of linked processes. For example: hypertext links can be created, can be passed around, and can disappear; the connections between cellular telephones and a network of base stations can change as the telephones are carried around. [...] The π-calculus [...] directly expresses movement of links in a space of linked processes.*

The celebrated works of **Xiuxiong Chen**, **Simon Donaldson**, and **Song Sun** (2015) ***Kähler-Einstein metrics on Fano manifolds. II: Limits with cone angle less than** 2π* and ***Kähler-Einstein metrics on Fano manifolds. III: Limits as cone angle approaches** 2π **and completion of the main proof*** are cited 215 and 206 times, respectively. In these papers, the authors study special Riemannian manifolds (M, g), where the so-called Kähler-Einstein metric g has specific conical singularities along a submanifold of real codimension 2. The cone angles in the titles refer to the angles of the conical singularities.

π appears the most in group theory

Group theory and generalizations is the subject in MSC2020 that has the most (633) publications with Π or π in the title. Interestingly, it seems that group theorists rarely read π as a real number, but rather a set of prime numbers or a group. In the following, we give some sample concepts in group theory that involve π in the usual notation.

Definition. Let G be a finite group and π a finite set of prime numbers. A π-*series* for G is a subnormal series (i.e., H_i is a normal subgroup of H_{i+1} for all $0 \leqslant i \leqslant n-1$) for G of the form $\{e\} = H_0 \leq H_1 \leq \cdots \leq H_n = G$, where each quotient H_{i+1}/H_i is either a π-group (i.e., all the prime factors of its order are in π) or a π'-group (i.e., none of the prime factors of its order are in π). If there exists a π-series for G, we say G is π-*solvable*.

Problem. Let π be a group. Construct a connected topological space $K(\pi, 1)$ such that $\pi_1(K(\pi, 1)) = \pi$ and $\pi_q(K(\pi, 1)) = 0$ for all other q.

Definition. A subgroup H of G is said to be π-*quasinormal* in G if it permutes with every Sylow subgroup of G.

Other subjects with many π-publications are number theory (351), computer science (256), and mathematical logic and foundations (203).

Photo References

We assume without loss of generality that the readers are able to find all above references in MathSciNet. Instead, we provide photos of all authors of the π-publications discussed in this poster.

Karl Schellbach and Cornelius Gouwens do not have a photo to the best of our knowledge.

Lindemann

Moore

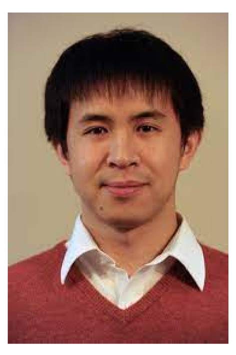
Skewes

Sangiorgi

Walker

Chen

Donaldson



Sun

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